and the $O(\delta^2)$ term in $\dot{\rho}_{1b}(t_1)$ given by

$$\frac{1}{V_1^2}\begin{pmatrix} 2c_0 + m_{11} \\ -d_0 + m_{21} \end{pmatrix}\frac{\delta^2}{2}$$

when $r_m(t)$ describes a circular orbit.

This completes the description of the computational technique used in first-order asymptotic matching to obtain the parameters of the moon centered hyperbola.

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Uncertainty in Measurement of Intermittency in Turbulent Free Shear Flows

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THE phenomenon of intermittency has come to be recognized as one of fundamental importance in turbulent free shear flows, although it has not been incorporated in turbulence theories to any extent. The intermittency factor (γ) has been measured in a variety of flows by various workers, e.g., Townsend in the wake of cylinders,[1] Corrsin and Kistler in boundary layers and round jets,[2] Bradbury in plane jets,[3] etc.

There are a number of methods of measuring γ. The direct method is to photograph the amplified hot-wire signal (either the u-component of turbulent fluctuations or its time derivative) and measure the duration of turbulent flow. The signal can be recorded on tape and analyzed by computer. An electronic circuit can be built which will indirectly measure the duration of turbulent bursts. This is the most common means of measuring γ. The purpose of this note is to focus attention on one of the inherent difficulties of using such a circuit, namely the calibration. This has been mentioned by Townsend[1] and by Corrsin and Kistler.[2] However, they have not given details regarding the extent of error that is involved. During the course of measurement of turbulent quantities in interacting wakes, the present authors have conducted a systematic analysis of this problem. The results are discussed below.

Figure 1 is a block diagram of the circuit that was used for measuring γ. The amplified, filtered, rectified output of the anemometer is lead into the Schmidt trigger. The output of the trigger is zero so long as the input is below a preset threshold value. This is the upper trip point (or UTP) when the input voltage exceeds the UTP the trigger output is a pulse of constant amplitude. This output continues so long as the input level is above a second trip point called the lower trip point (or LTP). When the input to the trigger falls below the LTP, the trigger output ceases. Because of the nature of the trigger design the two trip points cannot be made identical. However, they can be made sufficiently close that errors are not significant. This will be at the expense of the frequency response of the trigger. For present purposes this problem is not serious.

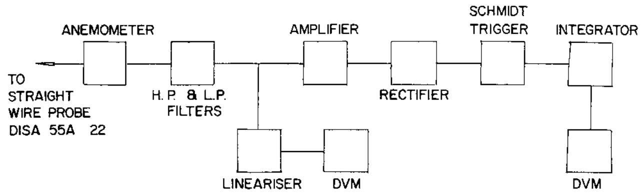

Fig. 1 Block diagram for measuring intermittency factor.

The main difficulty is in setting the threshold level or the UTP. This can be looked at from another point of view. Assuming the UTP has been fixed, to what level should the hot-wire signal be amplified? In the present investigation this view was adopted for the reason that it is possible to vary the amplification over a large range.

The hot-wire probe was located in the wake of a cylinder of 0.5 in. diam, at a downstream distance of 32.25 in. from the cylinder. The freestream velocity was held constant at about 87 fps. This gave a Reynolds number (based on the cylinder diameter) of 22,000. Keeping other factors constant, the amplifier gain was varied from about 150 to 320. At different values of gain, the integrator output was measured at three different locations (in the same plane). One point was along the wake symmetry axis ($y = 0$ in.), another at the edge of the wake ($y = 4$ in.) and the third in between ($y = 3.25$ in.).

The integrator output is shown plotted in Fig. 2 against the inverse of the amplification on semilog coordinates. It is seen that output varies exponentially with gain. The equation to these lines may be written as

$$\log E_{\text{int}} = (m/G) + \log C \tag{1}$$

where E_{int} is integrator output, G is amplification or gain, C is a constant and m is the slope of the lines. There are two points to be noted here: 1) C is nearly equal to the output of the integrator along the symmetry axis. This is to be expected since along the axis of the wake the flow is fully turbulent throughout the period of observation. Hence, the trigger would be fully open; 2) The slope of the lines, m, is a function of the intermittency factor at the location. This can be more clearly observed from Fig. 3. In this figure the integrator output at any point has been normalized

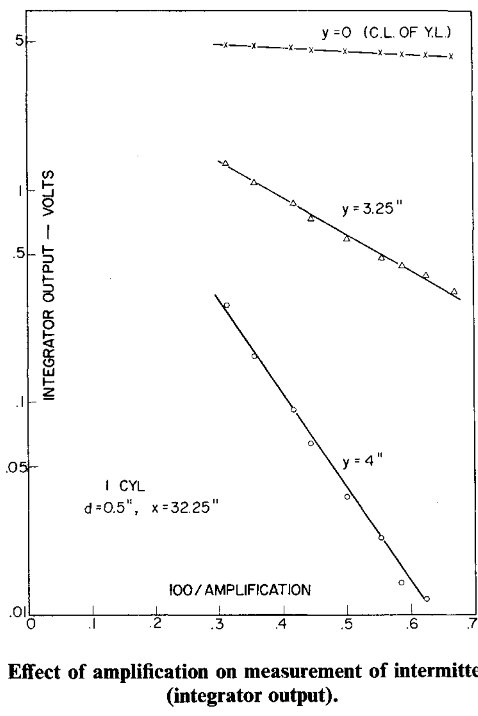

Fig. 2 Effect of amplification on measurement of intermittency factor (integrator output).

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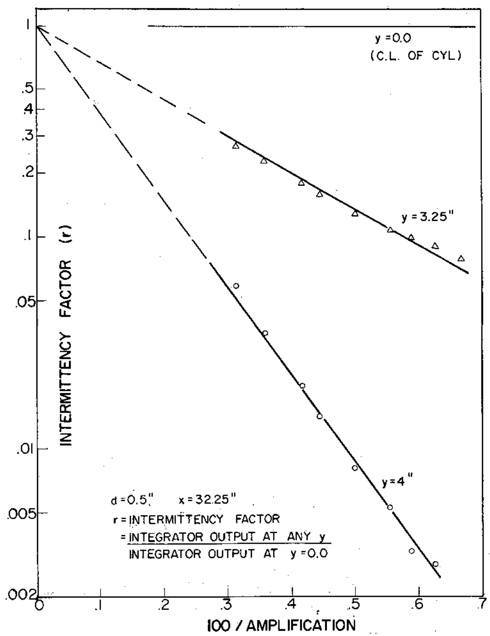

Fig. 3 Effect of amplification on measurement of intermittency factor (normalized integrator output).

by dividing it by the integrator output along the wake axis. This has also been plotted on semilog coordinates. It shows that the intermittency factor may be written as

$$\gamma(y) = \exp(m/G) \qquad (2)$$

This seems to arise from the fact that the effect of amplification on measured intermittency is dependent on the intensity of the turbulent bursts. With the same amplification or gain, the percentage increment of γ is lower for high-intensity turbulence than for low-intensity turbulence. This would explain the reduction in slope of the lines in Figs. 2 and 3 as one moves towards $y = 0$.

Regardless of the reasons for the variation of γ with amplification, it remains a fact that the calibration of such circuits is dependent on freestream turbulence level. In the present measurements the circuit was finally calibrated by photographing the rectified, amplified HW signal and measuring the duration of turbulent bursts. The difficulty with such a method is that it becomes progressively more difficult to distinguish between turbulent and nonturbulent regions in the area where $\gamma \approx 0.5$. This is illustrated by the present measurements in the wake of a cylinder of 0.5 in. diam at a distance 32.25 in. downstream. The HW signals were photographed at two points in the y direction. At $y/b = 1.21$, (where b is the nominal wake width given by Prandtl's mixing length theory) $\gamma = 0.102 + 2\%$. At $y/b = 0.95$, $\gamma = 0.487 \pm 12\%$. It is seen that the uncertainty in the calibration is quite high around $\gamma \approx 0.5$. Beyond $\gamma \approx 0.9$ the uncertainty again reduces.

It was attempted to correct the measurements by subtracting the integrator output when the probe was in the freestream from the measurements in the wake. This proved unsuccessful. Further details of the measurements are given in Ref. 4.

In conclusion, the present measurements indicate that when intermittency factor is measured using a circuit of the type described above, calibration by direct photography of the signals may be necessary. It further indicates that even with such calibration there might be errors of the magnitude of $\pm 10\%$ in certain sections of the measured profile.

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An Invalid Equation in the General Momentum Theory of the Actuator Disk

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THE general momentum theory of the actuator disk model of a propeller,[1] including a helicopter rotor, is incorrect. An unverified equation is included in the theory which, when combined with the other equations, leads to a contradiction. Specifically, the equation of axial momentum for the propeller states that the total thrust is expressible in terms of integrals of the wake parameters. Then the differential form of that equation is applied to the separate annular elements of the propeller. It is this application which leads to a contradiction. In fact, it was noted in Ref. 1, p. 187, that "The validity of this equation has not been established and its adoption may imply the neglect of the mutual interference between the various annular elements . . ."

The actuator disk model is the following. An incompressible, inviscid fluid with a uniform freestream axial velocity $V \geqq 0$, passes through an actuator disk of radius R. The actuator disk is normal to the freestream and is rotating at constant angular velocity Ω. The fluid, in passing through the disk, experiences a discontinuous increase in pressure p' and the angular velocity changes from zero to ω, while the axial component u and radial component v are continuous across the disk. The general nature of the flow behind the disk is shown in Fig. 1.

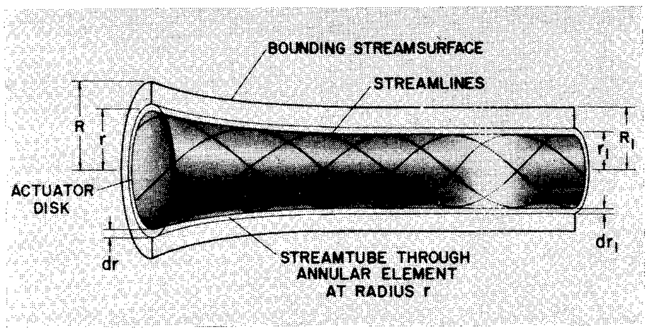

Fig. 1 Flow pattern.

As indicated in Fig. 1, r is the radial distance of any annular element of the propeller disk. In the final wake, let p_1 be the presssure, u_1 the axial velocity and ω_1 the angular velocity at a radial distance r_1 from the axis of the slipstream. p_0 is the pressure outside the slipstream and ρ the density.

Now from the equation of axial momentum for the propeller we have[1]

$$T = \rho \int_0^{R_1} u_1(u_1 - V)\, dS_1 - \int_0^{R_1} (p_0 - p_1)\, dS_1 \qquad (1)$$

This equation is generally accepted in the differential form

$$dT = \rho u_1(u_1 - V)\, dS_1 - (p_0 - p_1)\, dS_1 \qquad (2)$$

But we shall show that this equation is invalid.

The differential element of thrust at radius r is given by[1]

$$dT = \rho(\Omega - \tfrac{1}{2}\omega)\omega r^2\, dS \qquad (3)$$

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